We must build up a value of $\varphi_1(x)$ satisfying the last expression.

Take $e^{-\beta y}\sin\beta x$ and $e^{-\beta y}\cos\beta x$ and multiply the first by $\cos\beta\lambda$, the second by $\sin\beta\lambda$. Subtract these and we get $e^{-\beta y}\sin\beta(x-\lambda)$.

$$\therefore\ \varphi_1(x)=\frac{1}{\pi}\int_0^\infty\int_{-\infty}^\infty e^{-\beta y}\varphi(\lambda)\sin\beta(x-\lambda)d\beta d\lambda.$$

$$\text{Now }\int_0^\infty e^{-ax}\sin mx dx=\frac{m}{a^2+m^2}.$$

$$\therefore\int_0^\infty e^{-\beta y}\sin\beta(x-\lambda)d\beta=\frac{x-\lambda}{y^2+(x-\lambda)^2}.$$

$$\therefore\ \varphi_1(x)=\frac{1}{\pi}\int_{-\infty}^\infty\frac{(x-\lambda)\ \varphi(\lambda)d\lambda}{y^2+(x-\lambda)^2}.$$

$$\therefore f=vx+\frac{v}{\pi}\int_{-\infty}^\infty\frac{(x-\lambda)\ \varphi(\lambda)d\lambda}{y^2+(x-\lambda)^2}.$$

143. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Beads are fastened at equal intervals on a string placed over a smooth fixed pulley. If the original position of the string is one of symmetry, find the velocity at any moment, the pressure on the pulley, and the velocity with which the string leaves the pulley.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

In this solution the weights of the pulley and string are neglected.

Let $2l$=length of string, $2n+1$=number of beads each mass m.

Acceleration$=f=\dfrac{m[(n+x)-(n-x+1)]g}{m(2n+1)}=\dfrac{2x-1}{2n+1}g$, where x can have any value from 1 to $(n-1)$. l/n=distance between two beads.

$$\text{Now } l/n=\tfrac{1}{2}ft_1^2=\frac{(2x-1)gt_1^2}{2(2n+1)}.$$

$$\therefore t_1=\sqrt{\frac{2l(2n+1)}{gn(2x-1)}},\ v=ft_1=\sqrt{\frac{2gl(2x-1)}{n(2n+1)}}.$$

Let t be any time between the passing of the $(x+1)$th and $(x+2)$th particle over the pulley, then the velocity, v_1, at this time, is

$$v_1=\sum_{x=1}^{x=x}\sqrt{\frac{2gl(2x-1)}{n(2n+1)}}+\sqrt{\frac{2gl(2x+1)}{n(2n+1)}}t=\sqrt{\frac{2gl}{n(2n+1)}}[1+\sqrt{3}+\sqrt{5}+\sqrt{7}+\dots$$
$$+\sqrt{(2x-1)}+\sqrt{(2x+1)}t].$$

The velocity with which the string leaves the pulley

$$=\sum_{x=1}^{x=n-1}\sqrt{\frac{2gl(2x-1)}{n(2n+1)}}=\sqrt{\frac{2gl}{n(2n+1)}}[1+\sqrt{3}+\sqrt{5}+\ldots.+\sqrt{(2n-3)}].$$

Pressure on pulley at time $t=\dfrac{4m^2(n+x+1)(n-x)}{m(2n+1)}=\dfrac{4m(n+x+1)(n-x)}{2n+1}$

DIOPHANTINE ANALYSIS.

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92. Proposed by **M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find the sides of integral right triangles when the difference of the legs is given.

Solution by **HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.**

Let a be the difference in the legs, and x and $x+a$=legs. Then $2x^2+2ax+a^2=\square=$(say) $[px-a]^2=p^2x^2-2apx+a^2$. Whence, $x=\dfrac{2a[p+1]}{p^2-2}$. Take $p=2$, $x=3a$, and the sides are $3a$, $4a$, $5a$. Then in the formula $\dfrac{2[r+s]}{r+2s}$, we have $r/s=\frac{2}{1}$, then $p=\frac{3}{2}$, and $x=20a$, and the sides $20a$, $21a$, and $29a$, and so on *ad infinitum.*

Remark on Problem 94 by **HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.**

In his solution of this question, Professor Zerr gives up his general demonstration a little prematurely. It is true that "For integral values of m, m^2+m+1 is not a square," but fractional values of m lead to integral values of a, b, and c. The value of m which makes the expression a square is $\dfrac{2p+1}{p^2-1}$ in which p may be any number except one. Take $p=2$, and $m=\frac{5}{3}$. Then $a=\frac{40}{9}$, $b=\frac{24}{9}$, and $c=\frac{15}{9}$. Any common multiple of a, b, and c makes the square root of $\sqrt{[a^2+b^2+c^2]}$ a square and as the denominator is a square, makes abc a square, as well as these particular values. So a, b, and c may be taken=40, 24, and 15, respectively. Then $abc\sqrt{[a^2+b^2+c^2]}=14400=49$, a square number. However, the question does not call for integral values, and I had solved the question as follows from the point at which Professor Zerr leaves it. Substituting the value of $m=\dfrac{2p+1}{p^2-1}$ in the values of a, b, and c, reducing to common denominator, $[p^2-1]^2$, we have $a=p[2p^2+5p+2]$, $b=p[p+2][p^2-1]$, and $c=2p^3=p^2-2p-1$, in which p may be any number except one. Hence, there is an indefinite number of rational triangles whose area is a square.

96. Proposed by **L. C. WALKER, A. M., Graduate Student, Leland Stanford University, Cal.**

(*a*) Find the least three integral numbers such that the difference of every two of them shall be a square number; (*b*) find the least three square numbers such that the difference of every two af them shall be a square number.